Macroscopic parameters of Fokker-Planck flows

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ABSTRACT

The aim of this work is to investigate properties of solutions of Fokker - Planck equation in the context of continuum mechanics. We show that average quantities, calculated for these solutions approximately satisfy equations of isothermal motion of viscous ideal gas.

Keywords

Fokker-Planck equation, continuum mechanics

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1. Introduction

The statistical mechanics of systems described by kinetic equations and generalized Fokker - Planck equations is currently subject of active research (see [1]).

In this work we study rather special kind of medium without interaction of particles. To describe the state of this medium we use density distribution function $n(x_i, v_i)$ in the space of Cartesian coordinates x_i and corresponding velocities v_i.

We define average of any function f of these variables by

$$\bar{f}(x_i) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} n \ f \ dv_1 dv_2 dv_3. \tag{1}$$

Defined in such a way average value, depends obviously only on spatial coordinates x_i. Macroscopic observer deals with these average values and traits them from the continuum mechanics field theory point of view. He perceives average $\bar{f}$ as volume density of some additive function F

$$F = \iiint_D \bar{f} \, dx_1 dx_2 dx_3. \tag{2}$$

The most interesting functions are functions, which satisfy some conservation laws. That is time derivative of F is a sum of volume and surface integrals

$$\frac{dF}{dt} = \int\int_D\int \frac{\partial \bar{f}}{\partial t}\, dx_1 dx_2 dx_3 + \int\int_{\partial D} \overline{Fl}_1 dx_2 dx_3 + \overline{Fl}_2 dx_3 dx_1 + \overline{Fl}_3 dx_1 dx_2. \tag{3}$$

where $\overline{Fl}_i$ is the current of $\bar{f}$.

Our aim is to construct the set of conservation laws for special case, when distribution function $n(x_i, v_i)$ satisfies Fokker - Planck equation.

2. General definitions of macroscopic parameters

The density of mass is the average of unity

$$\bar{1} = \rho. \tag{4}$$

The current of mass vector is

$$\overline{v_i} = \rho u_i. \tag{5}$$

where u_i - average velocity.

(5) represents at the same time the vector momentum density.

The current of momentum tensor is

$$J_{ij} = \overline{v_i v_j}. \tag{6}$$

Decomposition of velocity gives

$$v_i = u_i + w_i; \tag{7}$$

where w_i - chaotic component of velocity. Macroscopic observer considers chaotic motion as heat. The average of chaotic component $\overline{w_i}$ is zero.

Proceeding with (6), we have

$$J_{ij} = \overline{(u_i + w_i)(u_j + w_j)} = \rho u_i u_j + \overline{w_i w_j}. \tag{8}$$

The tensor of stresses σ_{ij} is defined by

$$\sigma_{ij} = -\overline{w_i w_j}; \tag{9}$$

- recall classic kinetic gas theory formula for pressure.

Using this definition, we can write expression for the momentum current tensor in the final form

$$J_{ij} = \rho u_i u_j - \sigma_{ij}. \tag{10}$$

From macroscopical point of view the tensor of stresses describes the interaction between portions of continuum. It is remarkable, that though interaction between particles is missing, macroscopic cells are interacting nevertheless. This interaction is due to chaotic momentum transfer from one cell to another. For interacting particles additional term should arise in (9).

The volume density of energy is

$$e = \frac{1}{2}\,\overline{v_k v_k}; \tag{11}$$

because the only form of energy for non-interacting particles is their kinetic energy.

Decomposition (7) of velocities leads to decomposition of full energy

$$e = \frac{1}{2}\,\rho u_k u_k + \frac{1}{2}\,\overline{w_k w_k} = K + E. \tag{12}$$

The first term in (12) is kinetic energy density

$$K = \frac{1}{2}\,\rho u_k u_k. \tag{13}$$

The second term in (12) is energy of chaotic movement or internal energy volume density

$$E = \frac{1}{2}\,\overline{w_k w_k}. \tag{14}$$

For our simple model of non-interacting particles following relation between density of internal energy and pressure is valid

$$p = -\frac{1}{3}\,\sigma_{kk} = \frac{2}{3}\,E; \tag{15}$$

where p - the hydrostatic pressure.

The current of energy is

$$F_i = \frac{1}{2}\,\overline{v_k v_k v_i} = \frac{1}{2}\,\rho u_k u_k u_i + \frac{1}{2}\,\overline{w_k w_k} u_i + u_k \overline{w_k w_i} + \frac{1}{2}\,\overline{w_k w_k w_i} = \tag{16}$$

$$= (K + E)\;u_i - u_k \sigma_{ki} + \frac{1}{2}\,\overline{w_k w_k w_i}.$$

3. Expressions for macroscopic parameters through Fourier transform of density

Let us denote by M the Fourier transform of density

$$M(t, x_i, q_j) = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} n \exp(-i\;v_k q_k)\;dv_1 dv_2 dv_3. \tag{17}$$

where q_k - velocities momentum variables.

Using this definition, we can express macroscopic parameters in terms of Fourier transform of density and its derivatives:

- density

$$M(t, x_1, x_2, x_3, 0, 0, 0) = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} n dv_1 dv_2 dv_3 = \frac{\rho}{(2\pi)^3}\;; \tag{18}$$

- average velocity

$$\frac{\partial M}{\partial q_k}(t, x_1, x_2, x_3, 0, 0, 0) = \frac{-i}{(2\pi)^3}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} n v_k dv_1 dv_2 dv_3 = \frac{-i\rho u_k}{(2\pi)^3}\,; \tag{19}$$

- current of momentum tensor

$$\frac{\partial^2 M}{\partial q_i \partial q_j}(t, x_1, x_2, x_3, 0, 0, 0) = \frac{-1}{(2\pi)^3}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} n v_i v_j dv_1 dv_2 dv_3 = \frac{-J_{ij}}{(2\pi)^3}\,; \tag{20}$$

- volume density of energy

$$\frac{\partial^2 M}{\partial q_k \partial q_k}(t, x_1, x_2, x_3, 0, 0, 0) = \frac{-1}{(2\pi)^3}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} n v_k v_k dv_1 dv_2 dv_3 = \frac{-e}{(2\pi)^3}\,; \tag{21}$$

- current of energy is

$$\frac{\partial^3 M}{\partial q_i \partial q_k \partial q_k}(t, x_1, x_2, x_3, 0, 0, 0) = \frac{i}{(2\pi)^3}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} n v_i v_k v_k dv_1 dv_2 dv_3 = \frac{2iF_i}{(2\pi)^3}\,. \tag{22}$$

4. Fokker - Planck equation and conservation laws

Fokker - Planck equation with damping force is:

$$\frac{\partial n}{\partial t} + v_k\frac{\partial n}{\partial x_k} - \alpha\, v_k\frac{\partial n}{\partial v_k} - 3\,\alpha n = k\,\frac{\partial^2 n}{\partial v_m \partial v_m}\,; \tag{23}$$

where α - damping coefficient.

Multiplying (23) by $\exp(-i\ v_k q_k)$ and integrating over velocities, we obtain

$$\boxed{\frac{\partial M}{\partial t} + i\left(\frac{\partial^2 M}{\partial x_1 \partial q_1} + \frac{\partial^2 M}{\partial x_2 \partial q_2} + \frac{\partial^2 M}{\partial x_3 \partial q_3}\right) + \alpha\left(q_1\frac{\partial M}{\partial q_1} + q_2\frac{\partial M}{\partial q_2} + q_3\frac{\partial M}{\partial q_3}\right) = -k\left(q_1^2 + q_2^2 + q_3^2\right)M. \quad (24)}$$

Substitute $q_k = 0$ to (24), use (18-19) and get

$$\boxed{\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_k)}{\partial x_k} = 0. \quad (25)}$$

This is the mass conservation law.

Differentiate (24) by q_i, substitute $q_k = 0$, use (19-20) and get

$$\frac{\partial(\rho u_i)}{\partial t} + \frac{\partial J_{ij}}{\partial x_j} + \alpha\rho u_i = 0. \tag{26}$$

Use (10) and get

$$\boxed{\frac{\partial(\rho u_i)}{\partial t}+\frac{\partial(\rho u_i u_j)}{\partial x_j}-\frac{\partial \sigma_{ij}}{\partial x_j}+\alpha \rho u_i=0.} \qquad (27)$$

This is momentum conservation equation. To obtain another form of equation substitute $\partial\rho/\partial t$ from (12) to (13) and get

$$\boxed{\rho\frac{\partial u_i}{\partial t}+\rho u_j\frac{\partial u_i}{\partial x_j}-\frac{\partial \sigma_{ij}}{\partial x_j}+\alpha \rho u_i=0.} \qquad (28)$$

Differentiate (24) by q_j, q_k , substitute $q_k=0$ and get

$$\frac{\partial M_{jk}}{\partial t}+i\,\frac{\partial M_{ijk}}{\partial x_i}+2\alpha\ M_{jk}=-2k\ \delta_{jk}M. \qquad (29)$$

where $M_{ij}...$ - derivative of M on $q_i, q_j, ...$ in zero point.

Contract (29) on j,k and use (22-24)

$$\frac{1}{2}\frac{\partial J_{kk}}{\partial t}+\frac{\partial F_i}{\partial x_i}-\alpha\ J_{kk}=-3k\ \rho. \qquad (30)$$

$$\frac{\partial e}{\partial t}+\frac{\partial F_i}{\partial x_i}-2\alpha\ e=-3k\ \rho. \qquad (31)$$

Use expression (16) for F_i

$$\frac{\partial(K+E)}{\partial t}+\frac{\partial}{\partial x_i}\left((K+E)u_i-u_k\sigma_{ki}+\frac{1}{2}\overline{w_k w_k w_i}-2\alpha\ (K+E)\right)=-3k\ \rho. \qquad (32)$$

Contract equation (27) with u_i and get

$$\boxed{u_i\frac{\partial(\rho u_i)}{\partial t}+u_i\frac{\partial(\rho u_i u_j)}{\partial x_j}-u_i\frac{\partial \sigma_{ij}}{\partial x_j}+\alpha \rho u_i u_i=0.} \qquad (33)$$

5. Equation of state

The system of mass conservation law and momentum conservation equation is not closed. To close the system, we need express stresses through density, average velocities and their derivatives - equation of state.

For this purpose differentiate (24) on q_i and q_j and substitute $q_k=0$.

$$\frac{\partial M_{ij}}{\partial t}+i\,\frac{\partial M_{ijk}}{\partial x_k}+2\alpha M_{ij}=-k\delta_{ij}M. \qquad (34)$$

where M_{ij} - M derivatives in zero point; δ_{ij} - Kronecker delta.

Equation (34) contains the third derivative. If local velocities distribution is normal (Maxwell with nonzero average), then Fourier transform of this distribution is also normal

$$M = \exp\left(\frac{1}{2} A_{ij} q_i q_j + B_k q_k + C\right). \tag{35}$$

We take (35) as working hypothesis. To give it some justification, we refer to fundamental solution [1] of Fokker - Planck equation. It is obvious from physical considerations, that after sufficiently long time of evolution solution for arbitrary initial conditions asymptotically converges to fundamental solution with the same centroid and momentum.

If (35) holds, we get after simple calculations

$$M_{ijk} = \left(A_{ij} B_k + A_{jk} B_i + A_{ki} B_j + B_i B_j B_k\right) \exp(C). \tag{36}$$

Substitute (36) to (34) and get

$$\boxed{\frac{\partial \sigma_{ij}}{\partial t} + u_k \frac{\partial \sigma_{ij}}{\partial x_k} + \sigma_{ij} \frac{\partial u_k}{\partial x_k} + \sigma_{ik} \frac{\partial u_j}{\partial x_k} + \sigma_{kj} \frac{\partial u_i}{\partial x_k} + 2\alpha \sigma_{ij} = -2k\rho\delta_{ij}. \qquad (37)}$$

To obtain (37), we used (25), (27) and (28).

To simplify (37), we consider the static solution. In this case all velocities and their derivatives are zero and stresses are equal to hydrostatic pressure $\sigma_{ij} = -(k/\alpha)\rho\delta_{ij}$ - see below section 'Static solution of Fokker - Planck equation'. We consider approximately static solutions and use linearization method to obtain state equation for this case.

$$\sigma_{ij} = -\frac{k}{\alpha} \rho \delta_{ij} + \delta\sigma_{ij}. \tag{38}$$

Linearized form of (37) is

$$\frac{\partial \delta\sigma_{ij}}{\partial t} + 2\alpha \delta\sigma_{ij} = \rho \frac{k}{\alpha} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right). \tag{39}$$

We used the fact, that group of terms $(k/\alpha)[(\partial\rho)/(\partial t) + u_k(\partial\rho)/(\partial x_k) + \rho(\partial u_k)/(\partial x_k)]$ vanishes in consequence of the mass conservation law (25).

Equation (39) is ordinary differential equation with constant coefficients and variable righthand side. If righthand side changes sufficiently slow, solution of differential equation exponentially converges to

$$\delta\sigma_{ij} = \rho \frac{k}{\alpha^2} \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right). \tag{40}$$

Join (40) with (38) and find state equation

$$\boxed{\sigma_{ij} = -\frac{k}{\alpha}\rho\delta_{ij} + \rho\frac{k}{\alpha^2}\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right).} \tag{41}$$

We see, that (40) coincides with state equation of compressible viscous fluid. Relation between density and hydrostatic pressure is $p = (k/\alpha)\rho$.

It coincides with ideal gas isotherm. It was expected, because Fokker - Planck continuum is in thermal equilibrium with enclosing motionless medium.

Kinematic viscosity is equal to

$$\boxed{\nu = \frac{k}{\alpha^2}.} \tag{42}$$

6. Static solution of Fokker - Planck equation

In this section we briefly discuss the simple and clear case of static solution. Namely we calculate average quantities for uniform static solution of Fokker Planck equation

$$n = n_0 e^{-\frac{\alpha}{2k}v_m v_m}. \tag{43}$$

This is Maxwell distribution with zero average (see (47) below). Its Fourier transform is

$$M = M_0 e^{-\frac{k}{2\alpha}q_m q_m}; \tag{44}$$

where

$$M_0 = n_0\left(\frac{k}{\pi 2\alpha}\right)^{3/2}. \tag{45}$$

Density is equal to

$$\rho = (2\pi)^3 M_0 = n_0\left(\frac{2\pi k}{\alpha}\right)^{3/2}. \tag{46}$$

Average velocities are zero

$$u_k = 0. \tag{47}$$

The current of momentum tensor is

$$J_{ij} = (2\pi)^3 M_0 \frac{k}{\alpha}\delta_{ij} = \rho\frac{k}{\alpha}\delta_{ij}; \tag{48}$$

and as (49) holds

$$\rho u_i u_j = 0; \tag{49}$$

stresses are equal to

$$\sigma_{ij} = -(2\pi)^3 M_0 \frac{k}{\alpha} \delta_{ij} = -\rho \frac{k}{\alpha} \delta_{ij}. \tag{50}$$

Hydrostatic pressure is

$$p = (2\pi)^3 M_0 \frac{k}{\alpha} = \rho \frac{k}{\alpha}. \tag{51}$$

(51) is the equation of state and we easily identify it with equation of state of ideal gas. Comparison of (51) with original equation of state of ideal gas gives known relation

$$\frac{k}{\alpha} = k_B T; \tag{52}$$

where

k_B - Boltzmann's constant;

T - absolute temperature.

The volume density of energy is

$$e = 3/2(2\pi)^3 M_0 \frac{k}{\alpha} = 3/2\rho \frac{k}{\alpha} = 3/2\rho k_B T; \tag{53}$$

The kinetic energy density is equal to zero, because average velocities are zero. Therefore internal energy volume density is

$$E = 3/2(2\pi)^3 M_0 \frac{k}{\alpha} = 3/2\rho \frac{k}{\alpha} = 3/2\rho k_B T; \tag{54}$$

7. Fundamental solution of Fokker - Planck equation

In this section we discuss macroscopic properties of fundamental solution (see [2]). Fourier transform of fundamental solution on both space coordinates and velocities is

$$\boxed{\begin{aligned} N &= \frac{1}{(2\pi)^6}\exp\left\{-i\left[x_{0i}\,p_i + v_{0i}\left(\frac{p_i}{\alpha} + e^{-\alpha t}(q_i - \frac{p_i}{\alpha})\right)\right]\right\}\times \\ &\times\exp\left\{-k\left[\frac{t}{\alpha^2}\,p_i p_i + \frac{2}{\alpha^2}(1-e^{-\alpha t})\;p_i(q_i - \frac{p_i}{\alpha}) + \frac{1}{2\alpha}(1-e^{-2\alpha t})\,(q_i - \frac{p_i}{\alpha})\,(q_i - \frac{p_i}{\alpha})\right]\right\}. \qquad (55)\end{aligned}}$$

where p_i - variables, associated with space coordinates;
q_i - variables, associated with velocities;
and

$$\theta = 2\alpha t - (1 - e^{-\alpha t})\,(3 - e^{-\alpha t}). \tag{56}$$

To obtain expression for Fourier transform M only on velocities, we need another set of variables

$$\bar{x}_i = x_i - (x_{0i} + \frac{v_{0i}}{\alpha}(1 - e^{-\alpha t})). \tag{57}$$

Fourier transform M of fundamental solution on velocities is

$$\boxed{\begin{aligned} M &= \frac{1}{(2\pi)^3}\left(\frac{\alpha^3}{2\pi k\theta}\right)^{3/2}\exp(-ie^{-\alpha t}v_{0i}q_i)\times \qquad (58) \\ &\times\exp\left[-\frac{1}{\theta}\left(\frac{\alpha^3}{2k}\,\bar{x}_i\bar{x}_i + i\alpha(1-e^{-\alpha t})^2\;\bar{x}_i q_i + \frac{k}{\alpha}\left(\alpha t\,(1-e^{-2\alpha t}) - 2\,(1-e^{-\alpha t})^2\right)q_i q_i\right)\right].\end{aligned}}$$

Derivatives of M in zero point are

$$M_0 = \frac{1}{(2\pi)^3}\left(\frac{\alpha^3}{2\pi k\theta}\right)^{3/2}\exp\left(-\frac{1}{\theta}\,\frac{\alpha^3}{2k}\,\bar{x}_i\bar{x}_i\right). \tag{59}$$

$$M_i = \frac{-i}{(2\pi)^3}\left(\frac{\alpha^3}{2\pi k\theta}\right)^{3/2}\exp\left(-\frac{1}{\theta}\,\frac{\alpha^3}{2k}\,\bar{x}_k\bar{x}_k\right)\left(v_{0i}e^{-\alpha t} + \alpha(1-e^{-\alpha t})^2\,\frac{\bar{x}_i}{\theta}\right). \tag{60}$$

$$\begin{aligned} M_{ij} &= \frac{-1}{(2\pi)^3}\left(\frac{\alpha^3}{2\pi k\theta}\right)^{3/2}\exp\left(-\frac{1}{\theta}\,\frac{\alpha^3}{2k}\,\bar{x}_k\bar{x}_k\right)\times \qquad (61) \\ &\times\left[\left(v_{0i}e^{-\alpha t} + \alpha(1-e^{-\alpha t})^2\,\frac{\bar{x}_i}{\theta}\right)\left(v_{0j}e^{-\alpha t} + \alpha(1-e^{-\alpha t})^2\,\frac{\bar{x}_j}{\theta}\right) + \delta_{ij}\,\frac{2k}{\alpha\theta}\left(\alpha t(1-e^{-2\alpha t}) - 2\,(1-e^{-\alpha t})^2\right)\right].\end{aligned}$$

We get from (19) expression for density

$$\rho = (2\pi)^3 M_0 = \left(\frac{\alpha^3}{2\pi k\theta}\right)^{3/2} \exp\left(-\frac{1}{\theta}\,\frac{\alpha^3}{2k}\,\bar{x}_k \bar{x}_k\right). \tag{62}$$

After sufficiently long time we can drop exponentially small terms in (62) and get approximate solution

$$\theta' = 2\alpha t. \tag{56'}$$

$$x'_i = x_i - (x_{0i} + \frac{v_{0i}}{\alpha}). \tag{57'}$$

$$\rho' = (2\pi)^3 M_0 = \left(\frac{\alpha^3}{2\pi k\theta'}\right)^{3/2} \exp\left(-\frac{1}{\theta'}\,\frac{\alpha^3}{2k}\,x'_k x'_k\right). \tag{62'}$$

We see, that (62’) is fundamental solution of diffusion equation (when v_{0i} is independent from x_{0i}). So for sufficiently long times density approximately satisfies diffusion equation.

Average velocity vector is

$$u_i = \left(\frac{(2\pi)^3}{-i\rho}\right) M_i = v_{0i} e^{-\alpha t} + \alpha(1 - e^{-\alpha t})^2\,\frac{\bar{x}_i}{\theta}\,. \tag{63}$$

The first term in (63) is uniform translation. It’s velocity exponentially tends to zero. The second term is dilatation. After sufficiently long time the center of dilatation is (57’) and dilatation rate is $O(t)$ (see (56’)).

Stresses are

$$\sigma_{ij} = \rho u_i u_j + (2\pi)^3 M_{ij} = \left(-\frac{2k}{\alpha\theta}\right)\left(\alpha t(1 - e^{-2\alpha t}) - 2\,(1 - e^{-\alpha t})^2\right)\left(\frac{\alpha^3}{2\pi k\theta}\right)^{3/2} \exp\left(-\frac{1}{\theta}\,\frac{\alpha^3}{2k}\,\bar{x}_k \bar{x}_k\right)\delta_{ij}. \tag{64}$$

Strain rate tensor is

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) = \frac{\alpha}{\theta}\,(1 - e^{-\alpha t})^2 \delta_{ij}. \tag{65}$$

We see, that (41) holds, when we drop exponentially small terms.

In order to check momentum conservation law, we perform following calculations

$$\frac{\partial\rho}{\partial t} + \frac{\partial(\rho u_i)}{\partial x_i} = \tag{66}$$

$$= \rho\left(-\frac{3}{2}\frac{1}{\theta}\frac{\partial\theta}{\partial t}+\frac{1}{\theta^2}\frac{\partial\theta}{\partial t}\frac{\alpha^3}{2k}\bar{x}_k\bar{x}_k+\frac{\alpha^3}{\theta k}\bar{x}_k\frac{\partial\bar{x}_k}{\partial t}\right)+\rho\left(-\frac{\alpha^3}{\theta k}\bar{x}_k(v_{0k}e^{-\alpha t}+\alpha(1-e^{-\alpha t})^2\frac{\bar{x}_k}{\theta})+\alpha(1-e^{-\alpha t})^2\frac{1}{\theta}\right).$$

where

$$\frac{\partial\theta}{\partial t}=2\alpha(1-e^{-\alpha t})^2; \qquad (67)$$

and

$$\frac{\partial\bar{x}_i}{\partial t}=-v_{0i}e^{-\alpha t}. \qquad (68)$$

This proves (27) for our special case.

$$\frac{\partial(\rho u_i)}{\partial t}+\frac{\partial(\rho u_i u_j)}{\partial x_j}-\frac{\partial\sigma_{ij}}{\partial x_j}+\alpha\rho u_i= \qquad (69)$$

$$=\rho\left(-\alpha v_{0i}e^{-\alpha t}+2\alpha^2(1-e^{-\alpha t})\frac{\bar{x}_i}{\theta}e^{-\alpha t}-\alpha(1-e^{-\alpha t})^2\frac{v_{0i}}{\theta}e^{-\alpha t}-\alpha(1-e^{-\alpha t})^2\frac{\bar{x}_i}{\theta^2}2\alpha(1-e^{-\alpha t})^2\right)+$$

$$+\rho\left(v_{0i}e^{-\alpha t}+\alpha(1-e^{-\alpha t})^2\frac{\bar{x}_i}{\theta}\right)\alpha(1-e^{-\alpha t})^2\frac{1}{\theta}-\rho(-\frac{\alpha^3}{\theta k}\bar{x}_i)(-\frac{2k}{\alpha\theta})\left(\alpha t(1-e^{-2\alpha t})-2\,(1-e^{-\alpha t})^2\right)+$$

$$+\alpha\rho\left(v_{0i}e^{-\alpha t}+\alpha(1-e^{-\alpha t})^2\frac{\bar{x}_i}{\theta}\right)=$$

$$=\rho\frac{\bar{x}_i}{\theta}\left(\frac{\alpha^2}{\theta}(1-e^{-\alpha t})^4-2\frac{\alpha^2}{\theta}\left(\alpha t(1-e^{-2\alpha t})-2\,(1-e^{-\alpha t})^2\right)+\alpha^2(1-e^{-\alpha t})^2+2\alpha^2(1-e^{-\alpha t})e^{-\alpha t}-2\frac{\alpha^2}{\theta}(1-e^{-\alpha t})^4\right)=$$

$$=\rho\frac{\bar{x}_i}{\theta}\left(-\frac{\alpha^2}{\theta}(1-e^{-\alpha t})^4-2\frac{\alpha^2}{\theta}\left(\alpha t(1-e^{-2\alpha t})-2\,(1-e^{-\alpha t})^2\right)+\alpha^2(1-e^{-2\alpha t})\right)=$$

$$=\rho\frac{\bar{x}_i}{\theta}\frac{\alpha^2}{\theta}\left(-(1-e^{-\alpha t})^4-2\alpha t(1-e^{-2\alpha t})+4\,(1-e^{-\alpha t})^2+(1-e^{-2\alpha t})(2\alpha t-(1-e^{-\alpha t})(3-e^{-\alpha t}))\right)=$$

$$=\rho\frac{\bar{x}_i}{\theta}\frac{\alpha^2}{\theta}(1-e^{-\alpha t})^2\left(-(1-e^{-\alpha t})^2+4-(1+e^{-\alpha t})(3-e^{-\alpha t})\right)=0.$$

Another conservation laws we can check in the similar way.

DISCUSSION

We obtain equations of motion of Fokker - Planck continuum. Local average velocities and stresses, calculated for Fokker - Planck equation solutions, satisfy these equations.

With use of non strict arguments we deduct, that state equation of Fokker - Planck continuum is approximately state equations of isothermal viscous compressible fluid.

We obtain relation between kinematic viscosity of this fluid and coefficients of Fokker - Planck equation - coefficient of damping and coefficient of diffusion in velocities space.

In the last sections macroscopic parameters of Fokker-Planck flows for static solution and fundamental solution are calculated.

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